Sample Problems, Set 1

1. **Minimal Two-Level Forms**

   Provide minimal sum-of-products and product-of-sums expressions for the following functions.

   (a) $f(a,b,c,d) = \bar{a}bc\bar{d} + \bar{a}bcd + abc\bar{d} + ab\bar{c}d + a\bar{b}cd + \bar{a}\bar{b}c\bar{d} + \bar{a}\bar{b}\bar{c}d + a\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}cd + abcd + ab\bar{c}\bar{d}a\bar{b}c\bar{d}$

   (b) $g(a,b,c,d) = a\bar{b}cd + a\bar{b}\bar{c}d + a\bar{b}\bar{c}\bar{d} + ab\bar{c}\bar{d} + abc\bar{d} + \bar{a}bcd + \bar{a}\bar{b}cd + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}\bar{d} + abcd$

   (c) $z(a,b,c,d,e,f) = abcdef + abcde\bar{f} + abcd\bar{e}\bar{f} + abc\bar{d}\bar{e}\bar{f} + ab\bar{c}\bar{d}\bar{e}\bar{f} + a\bar{b}\bar{c}\bar{d}\bar{e}\bar{f} + a\bar{b}c\bar{d}e\bar{f} + ab\bar{c}de\bar{f} + \bar{a}\bar{b}cd\bar{e}\bar{f} + a\bar{b}\bar{c}de\bar{f} + a\bar{b}\bar{c}\bar{d}ef + a\bar{b}\bar{c}\bar{d}\bar{e}f + \bar{a}bcde\bar{f} + \bar{a}bcd\bar{e}\bar{f} + ab\bar{c}de\bar{f} + abcd\bar{e}f$

2. **More Minimal Two-level Forms**

   Consider the example shown in Figure 1: a 7-segment display decoder. The inputs are four bits, x_0, x_1, x_2, x_3, specifying a number in binary. The outputs are 7 bits, a, b, c, d, e, f, g, specifying which segments to light up in a display – such as that of a digital alarm clock – to form the image of the corresponding decimal number. Our goal is to design a circuit that implements the functions a, b, c, d, e, f, g (with 0 corresponding to "off" and 1 corresponding to "on").

   Since the four bits x_0, x_1, x_2, x_3 specify numbers from 0 to 15, but the display is only for 0 through 9, we'll adopt the convention that for inputs corresponding to 10, 11, 12, 13, 14 and 15 *all* the segments should light up (i.e., a through g evaluate to 1 for these inputs).

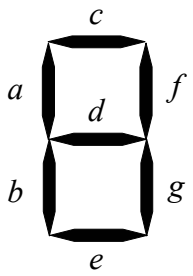

Figure 1: 7-Segment Display.

Fill in the following truth table.

inputs				outputs						
x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0							
0	0	0	1							
0	0	1	0							
0	0	1	1							
0	1	0	0							
0	1	0	1							
0	1	1	0							
0	1	1	1							
1	0	0	0							
1	0	0	1							
1	0	1	0							
1	0	1	1							
1	1	0	0							
1	1	0	1							
1	1	1	0							
1	1	1	1							

Find minimal sum-of-product and product-of-sums expressions for a, b, c, d, e, f and g.

3. **Timing Analysis**

For the circuits in Figures 2, 3 and 4, compute the arrival times for each output function for each input combination.

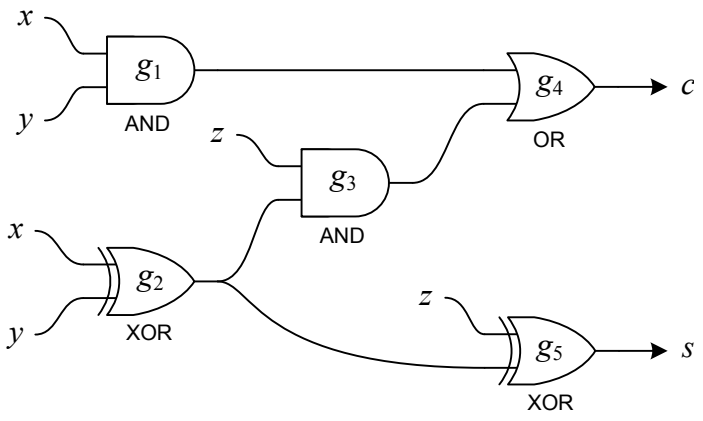

Figure 2: Circuit

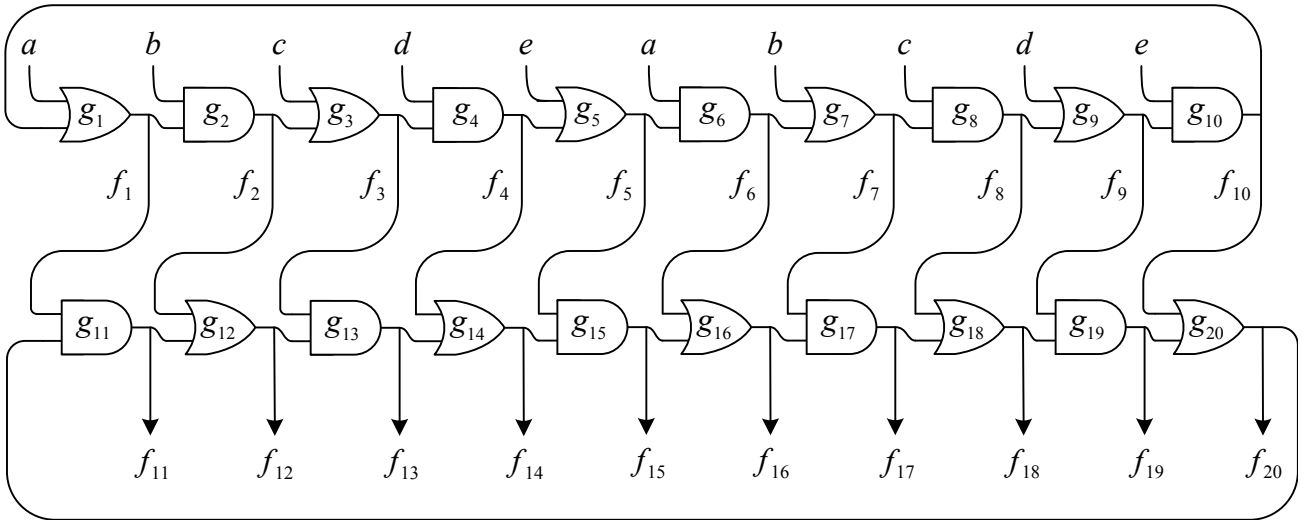

Figure 3: Circuit

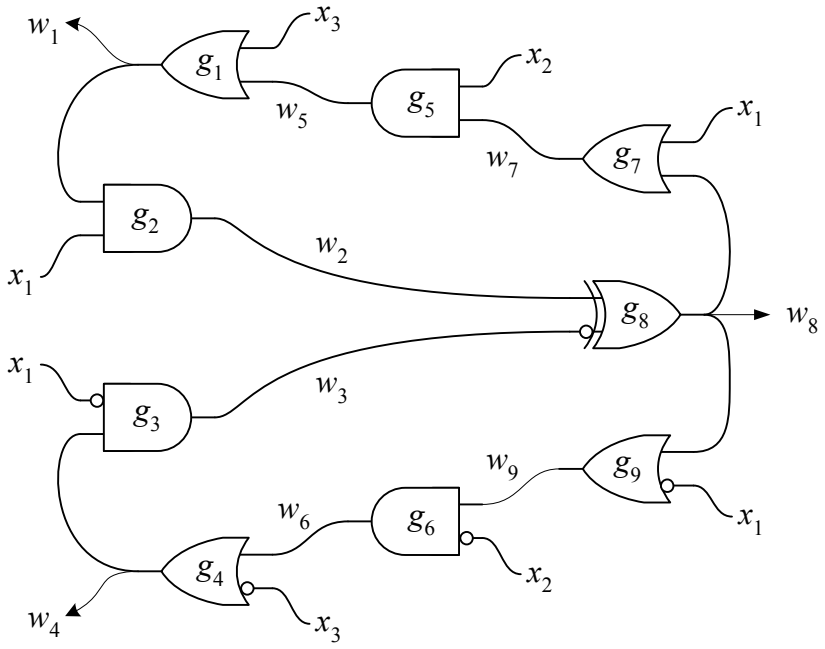

Figure 4: Circuit